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How to do Subcycled, Self-Gravitating Hydrodynamics on an Adaptive Mesh: Essential Obstacles and Fundamental Methodology

R. Fisher, R. Klein, L. Howell, J. Greenough, C. McKea

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How to do subcycled, self-gravitating hydrodynamics on an adaptive mesh: essential obstacles and fundamental methodology

Robert Fisher (LLNL)
Richard Klein (LLNL/UCB)
Louis Howell (LLNL)
Jeff Greenough (LLNL)
Christopher McKee (UCB)

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ASCI Flash Center
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Overview

- Fundamental equations
- AMR Background: Hydro dynamics
- Option 1: Dirichlet BCS
- Option 2: Neumann BCS
- Test Results

Fundamental Equations

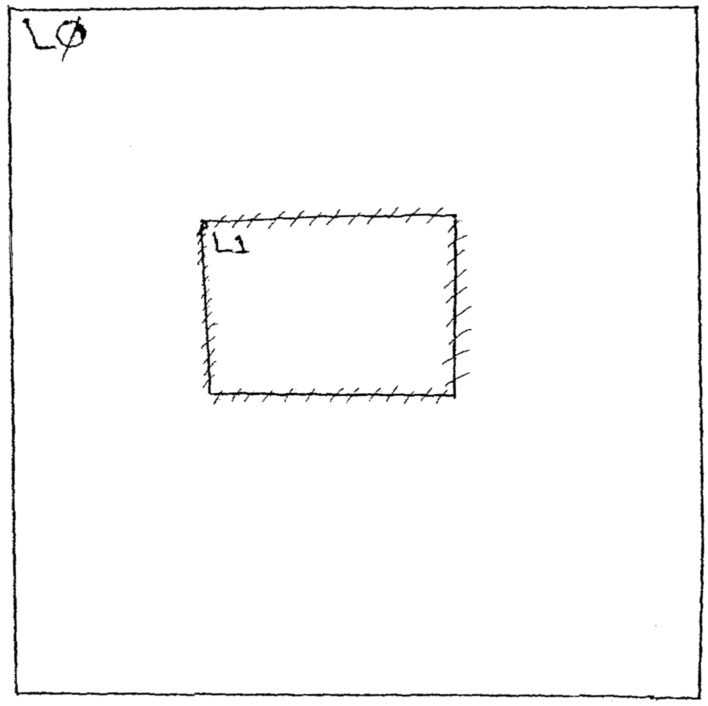
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial (\rho \vec{v})}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} \vec{v}) = - \vec{\nabla} P - \rho \vec{\nabla} \Phi$$

$$\vec{\nabla}^2 \Phi = 4\pi G \rho$$

$$\left. P = (\gamma - 1) \rho \, \varepsilon_{int} \right\} \text{ in this work}$$

AMR Background: Hydrodynamics



1) Advance level 0 at timestep Δt_0.

2) Advance level 1 at timestep $\Delta t_1 = \Delta t_0 / \text{ref}$.

- Use time- and space-interpolated values for ghost cells on $\partial(L\emptyset)$.

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3) Coarsen down L1 data onto LØ.

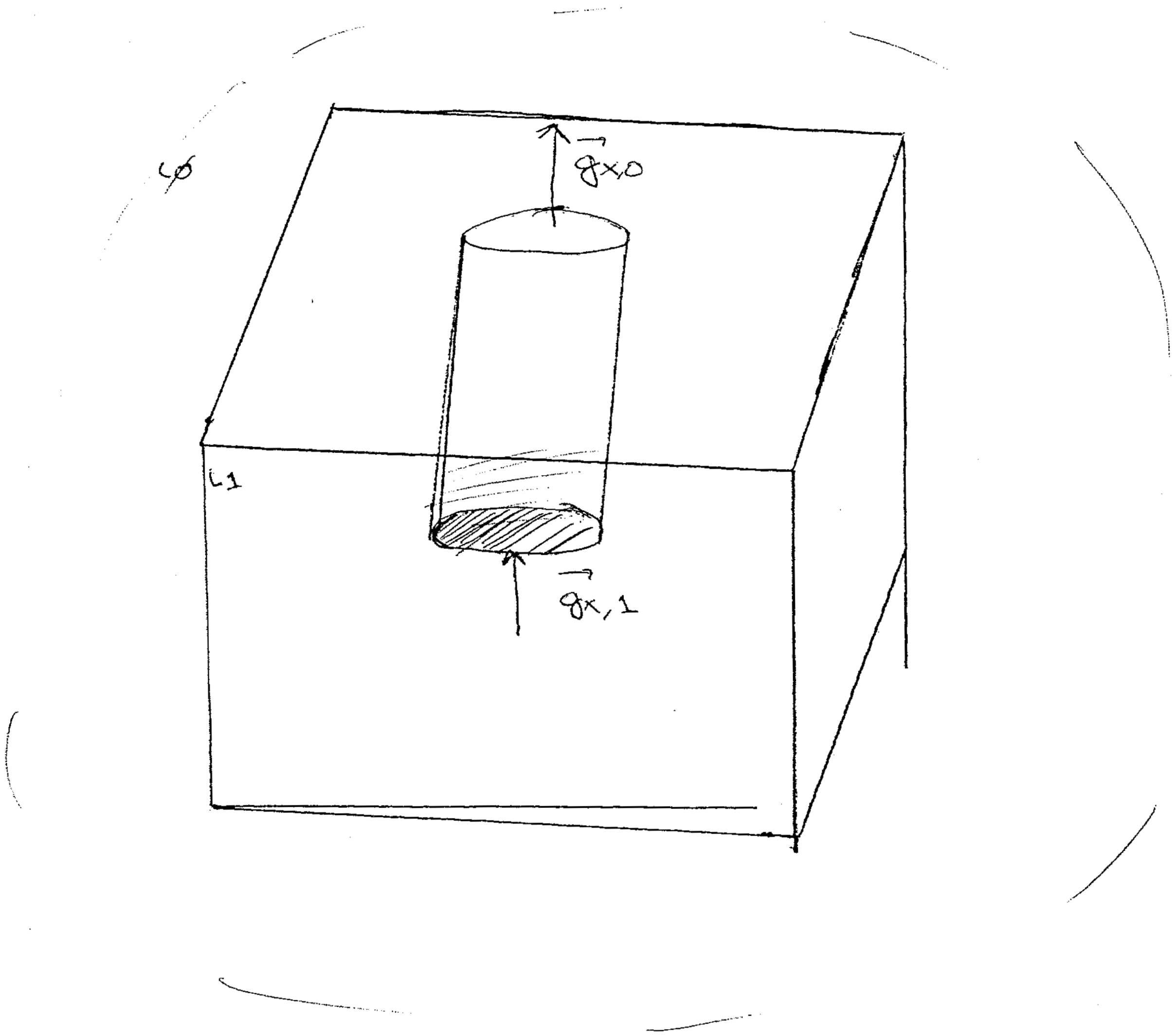
4) Correct for ~~flux~~ flux mismatches via refluxing.

Option 1: Dirichlet bcs.

Assign $\Phi(\vec{x},t)$ on grid boundaries using Dirichlet.

$$\Phi(\vec{x},t) = \mathcal{S}(\vec{x},t) \Big\} \text{ where } \vec{x} \in \partial(L1)$$

$\Longrightarrow \Phi$ continuous, $\vec{\nabla}\Phi$ not.



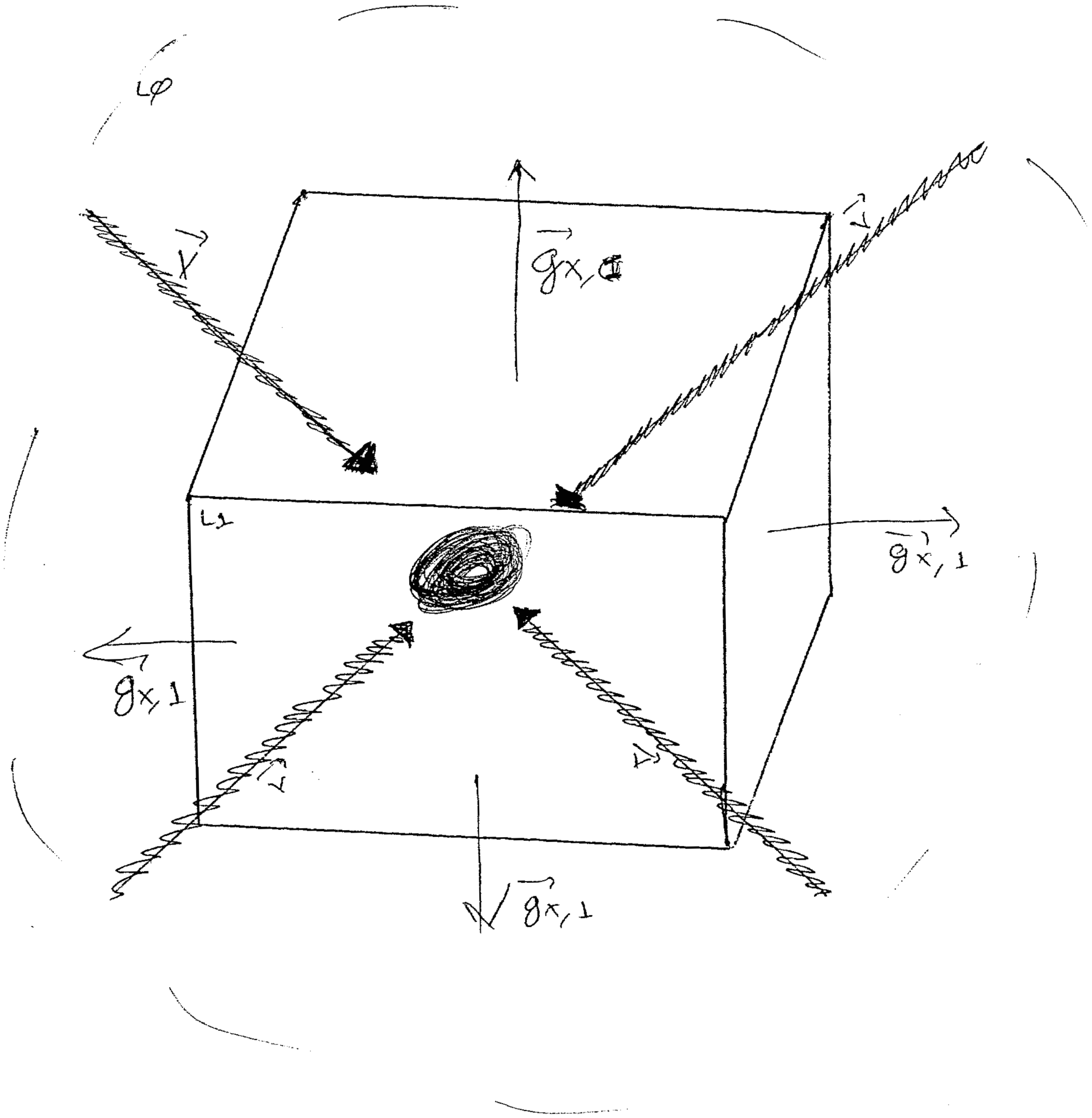
$$\hat{n} \cdot (\vec{g}_{x,0} - \vec{g}_{x,1}) = -4\pi G \Sigma$$

Option 2: Neumann bcs

Assign $\vec{\nabla}\Phi(\vec{x},t)$ on grid boundaries using Neumann.

$$\left.\vec{\nabla}\Phi(\vec{x},t) = \vec{g}(\vec{x},t)\right\} \text{ where } \vec{x} \in \partial(L1)$$

$\Longrightarrow$ $\vec{\nabla}\Phi$ continuous, Φ not.



$$\int_A \cancel{dA}\ \vec{g} \cdot d\vec{A} = 4\pi G M_{enclosed}$$

Algorithm

If ($l < l_{max}$) and (regrid requested for timestep n) **Then**

For $l' \in \{l_{max}-1,\ldots,l\}$ **do**

- Determine new grid layout for level $l'+1$.
- Interpolate data to new grids from level l'.
- Copy data on intersection with old level $l'+1$ grids.

End do

If ($l < l_{max}$ or $t^{old,l} = t^{old,l-1}$) **then**

On $\partial\Omega^l$ and $P(\partial\Omega^{l+1})$:

- Store $F_{rad}^{l,n}$ and $\vec{g}^{l,n} = -\nabla\Phi^{l,n}$ in registers.

On Ω^l :

- Compute $\nabla^2\Phi^{l,n}$ and $\nabla \cdot F_{rad}^{l,n}$.

Endif

Level Time Step, level l :

- **Multi-Fluid Explicit Godunov Hydrodynamic Update**
- **FLD Radiative Level Solution and Implicit Energy Update**
- **Poisson Level Solution and Explicit Momentum and Energy Update**

End Level Time Step

If $l < l_{max}$ **then**

FLD Radiation Multilevel Solve: $l' \in l,\ldots,l_{max}$

Poisson Multilevel Solve: $l' \in l,\ldots,l_{max}$ (Solve for Φ_{corr} and $\vec{g}_{corr}$)

- $D^{E\rightarrow C}(\vec{G}^{C\rightarrow E}\Phi_{corr}) = \tilde{D}^{E\rightarrow C}(\delta\vec{g}^{l+1,n+\theta})$

- $\vec{g}_{corr} = -\vec{G}^{C\to C}\Phi_{corr}$

End Poisson Multilevel Solve

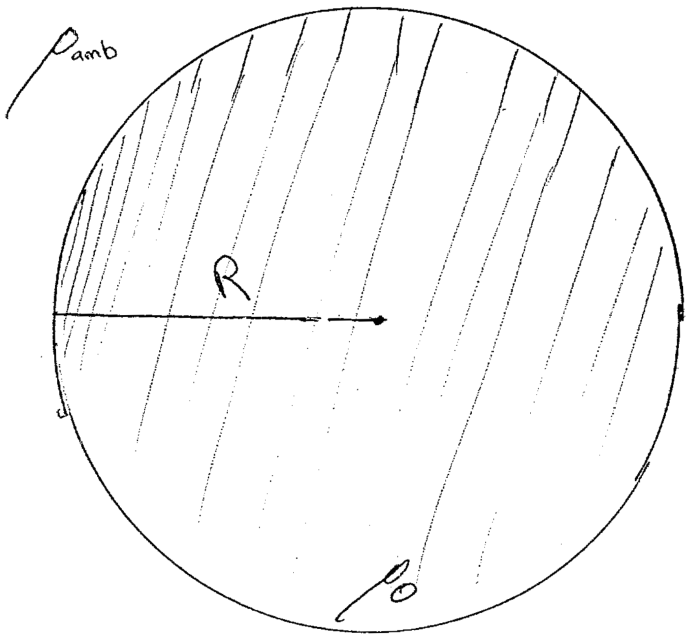
For $l' \in l_{max}, \ldots, l$ do

On $\Omega^{l'+1} - \mathrm{P}\,(\Omega^{l'+1})$:

- $(\rho\vec{v})^{l',n+1} = (\rho\vec{v})^{l',n+1} + \rho^{l',n+1}\vec{g}_{corr}^{l',n+1}\Delta t^{l'}$
- $\Phi^{l',n+1} = \Phi^{l',n+1} + \Phi_{corr}^{l',n+1}$

End do

Test Problems

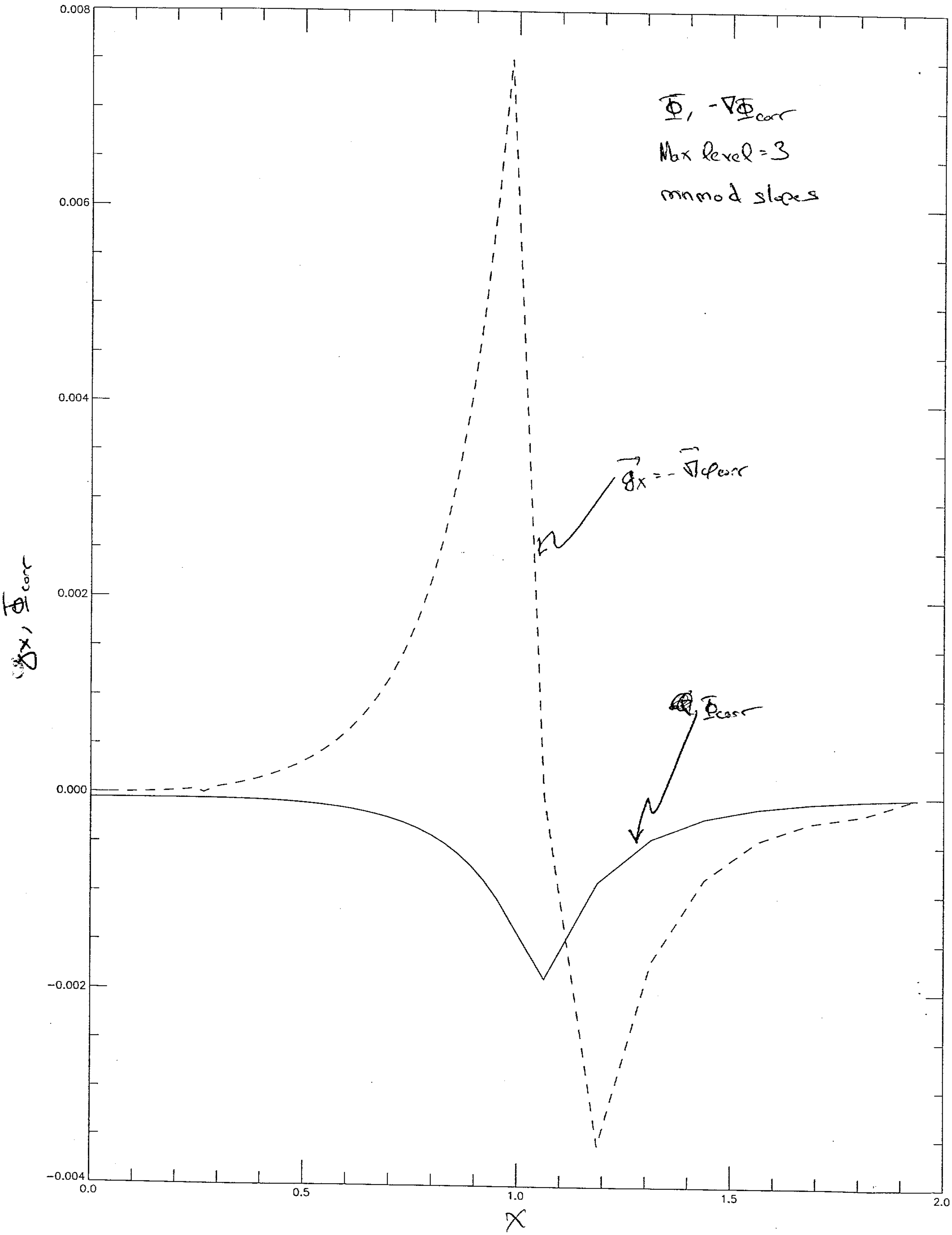


$$\chi_\rho = \rho_0 / \rho_{amb}$$

$$\chi_p = P_0 / P_{amb}$$

$$\alpha = \frac{\text{thermal energy}}{|\text{gravitational energy}|}$$

$$\beta = \frac{\text{rotational energy}}{|\text{gravitational energy}|}$$


0.008
0.006
0.004
0.002
0.000
−0.002
−0.004
0.0
0.5
1.0
1.5
2.0
Φ, $-\nabla\Phi_{corr}$
Max level = 3
minmod slopes
$\vec{g}_x = -\vec{\nabla}\phi_{corr}$
Φ_{corr}
g_x, Φ_{corr}
x

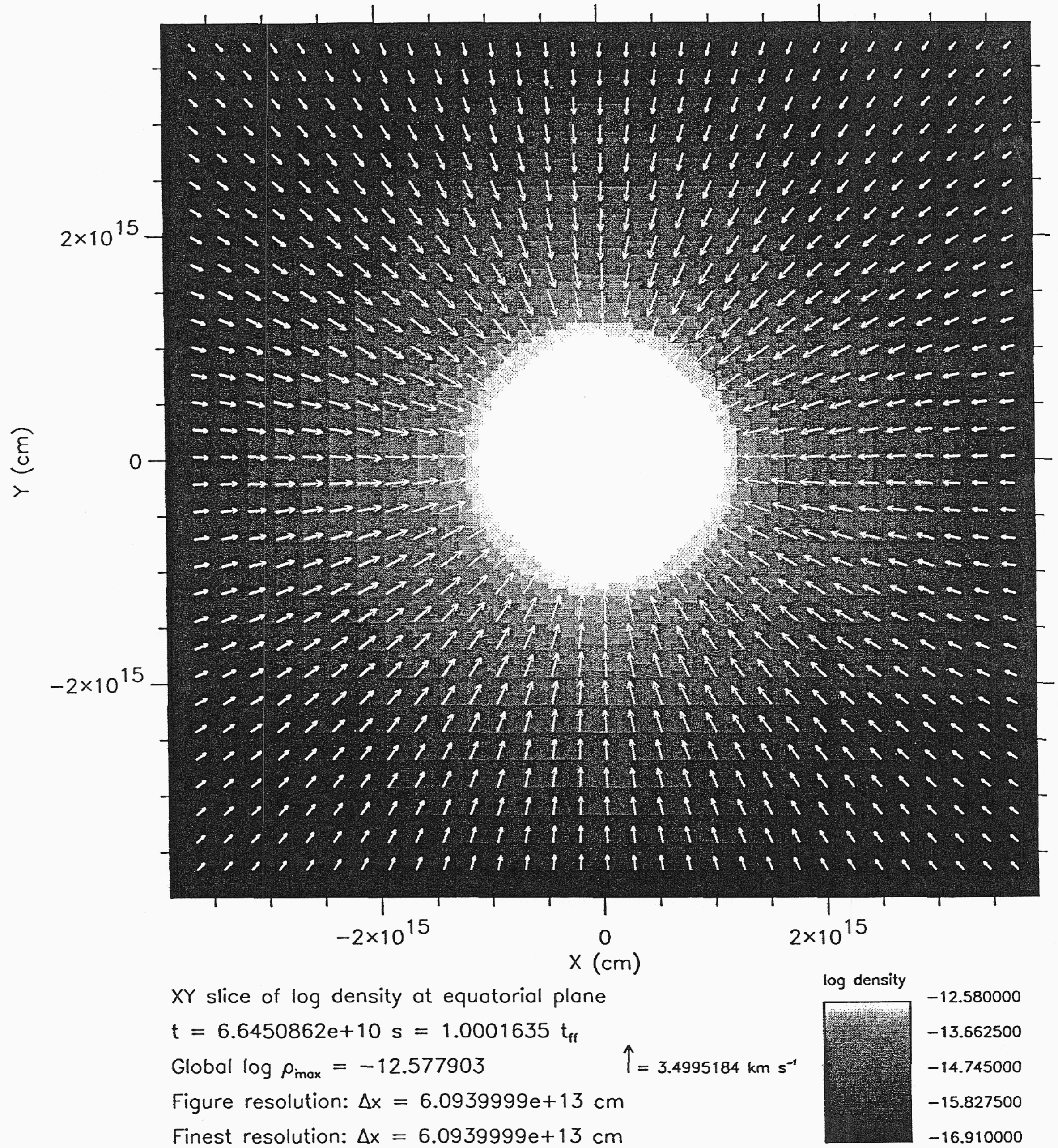


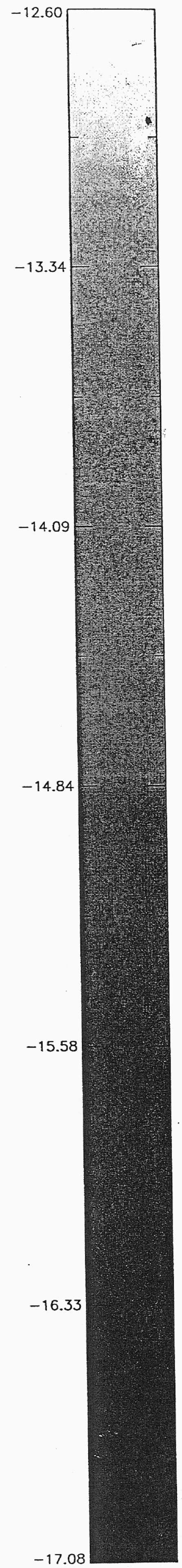
FIG. 5.—Equatorial slice of log density in the uniform collapse problem. At the time shown, about half of the cloud mass, gas inside the central plateau in this figure, remains undisturbed by the inward-moving rarefaction wave.

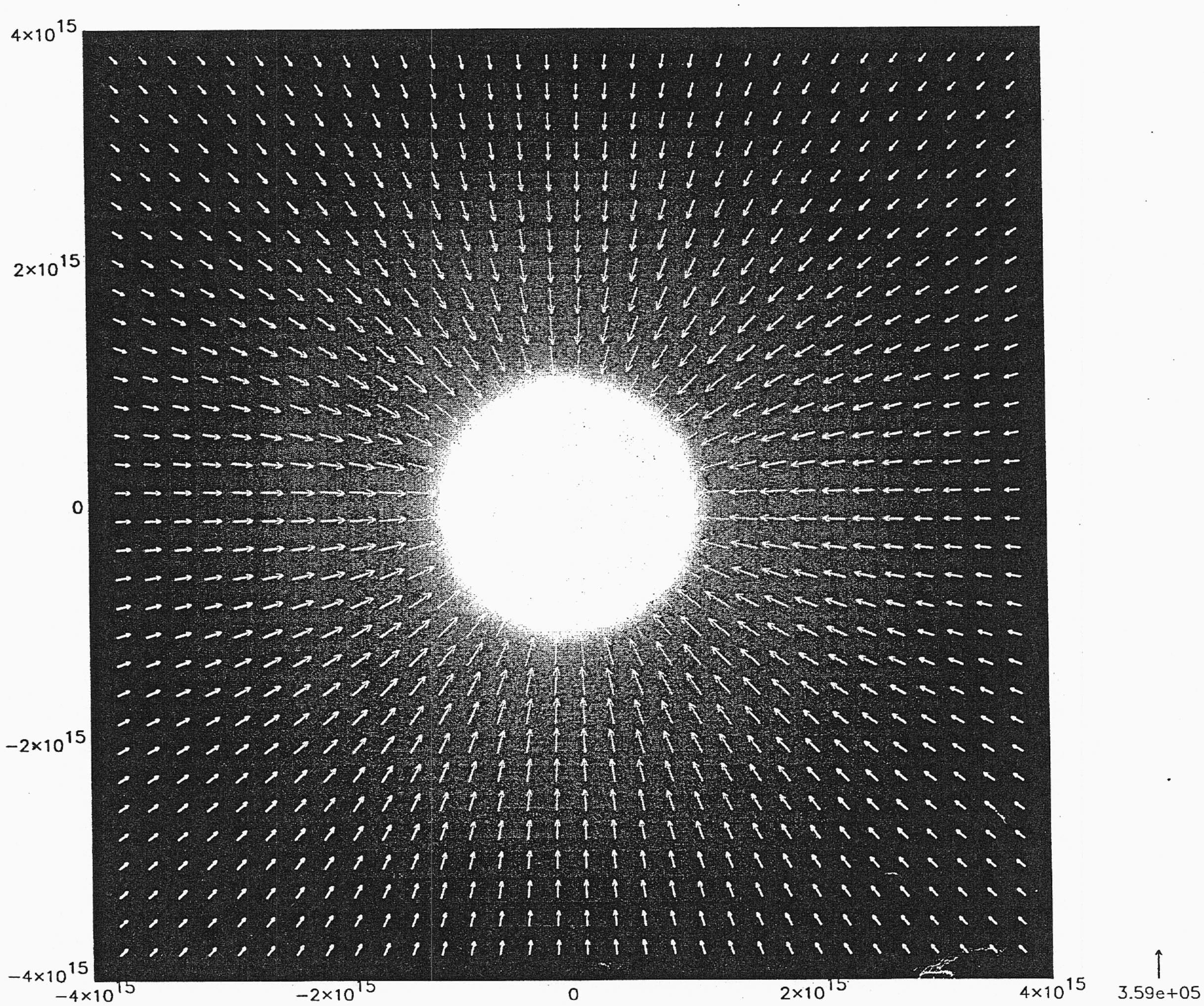
the result that the grids form simple cubes about the cloud. Tracing a radial line, we therefore first encounter decreased resolution along the coordinate axes; this is clearly visible in Figure 5. Because the gradient is most poorly resolved at these points, the volume averaging of the highest density gas just outside the finer grid is most approximate at these six (in three dimensions) cloud-grid tangent points. This, too, can be seen in the Figure 5 as distinct (though slight) overdense regions (relative to gas at the same radii) at these points in the equatorial plane. An increase in χ_P leads to an increase in the rate of expansion of the surface layers of the cloud and a hastened reduction of the steep gradient, resulting in a lowering of the effective χ_ρ. We found that use of $\chi_P = 10$ reduced the bump amplitude relative to the plateau by 40% in the log when compared to the $\chi_P = 1$ run at the same $\rho_{\max} = 10^{-11.6}$ g cm^{-3}. In runs of uniform clouds with more astrophysically realistic energy ratios α and β_Ω, such as the remainder of the uniform clouds considered in this paper, we did not find this edge effect to be of importance. An increase in either could be expected to damp the perturbations. We conclude that it is a numerical artifact of little practical significance and is realized only at the extremely low α and β_Ω of this problem. It does, however, 6/19/2003

$J = 1/8$, R_{64} Cold Uniform Collapse

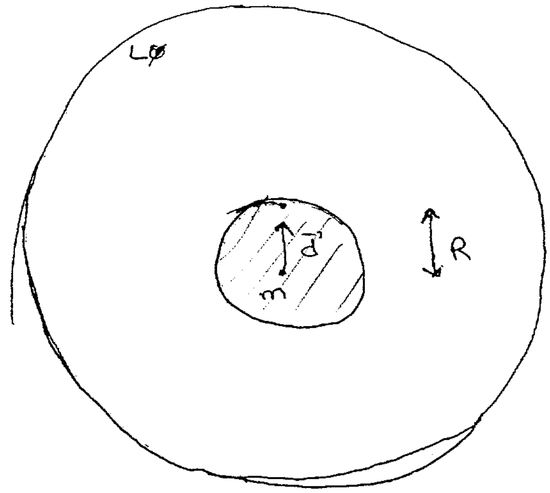
Refinement Factor 2

$t = 6.51399 \times 10^{10}\,s = .9805\, t_{ff}$





Further Considerations



$$4\pi G \Sigma = \left(-\vec{\nabla}\varphi_0 + \vec{\nabla}\varphi_1 \right)_{r=R} = -\vec{\nabla}\left(-\frac{G\vec{p}\cdot\vec{r}}{\vec{r}^2} \right)_{r=R}$$

$$= \left(-\frac{2Gm\,d\cos\theta}{r^3} \right)_{r=R} = \frac{-2Gm\,d\cos\theta}{R^3}$$

$$\boxed{\Sigma = \frac{-m}{2\pi}\,\frac{d}{R^3}\,(\cos\theta)}$$